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Isobaric analogue states in nuclei

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Abstract. The masses within an isobaric multiplet are related by the general model-independent mass formula

$$M(A, T, T_z) = a(A, T) + b(A, T)T_z + c(A, T)T_z^2.$$

The values of the coefficients $b(A, T)$ and $c(A, T)$ are extracted from experimental data as a function of mass number for several triplets ranging from $A = 6$ to $A = 32$. Theoretical calculations are made for these coefficients on the basis of Coulomb interaction alone after accounting for the neutron–proton mass difference, and the residual shifts are attributed to charge-dependent effects in the nuclear forces. A phenomenological effective charge-dependent potential is introduced and its parameters are determined by a least-squares analysis.

1. Introduction

The mass splitting between members of an isobaric multiplet is due to the neutron–proton mass difference, Coulomb interaction and possible departures from charge independence. A source of information about the charge independence of nuclear forces in the past has been the comparison of the energy levels of isobaric triplets which has been made by Wilkinson (1956), Fairbairn (1961) and Sengupta (1962). They have assumed that the nuclear forces are charge symmetric and that for a homogeneous charge distribution the Coulomb energy follows the simple $A^{-1/3}$ law. Now the masses within an isobaric multiplet are related by the general model-independent formula

$$M(A, T, T_z) = a(A, T) + b(A, T)T_z + c(A, T)T_z^2.$$

It was pointed out by Wilkinson (1964) that this formula holds even if the nuclear forces are charge dependent and this charge dependence can be treated as a perturbation. Convincing evidence for the presence of charge-dependent effects of non-Coulomb origin has been given by Blin-Stoyle *et al.* (1964) and Yap (1967). They have obtained good agreement with experimental values of the Fermi matrix element when considering a short-range charge-dependent nucleon–nucleon potential in addition to the usual Coulomb potential.

In this paper a number of isobaric triplets ranging from $A = 6$ to $A = 32$ are analysed. The excitation energy for states with $T = 1$ has been measured precisely in several self-conjugate ($T_z = 0$) nuclei. In § 2 these data are used to determine the b and c coefficients in the mass formula as a function of mass number. The coefficients are then calculated by assuming that the Coulomb interaction between the protons and the neutron–proton mass difference are the two factors contributing to the mass splitting in each triplet. In § 3 charge-dependent effects are introduced. The values of the parameters involved in an effective phenomenological charge-dependent potential which could account for the observed values of $b(A, T)$ and $c(A, T)$ are determined by a least-squares analysis. In § 4 the results and limitations of the above analysis are discussed and we conclude that the inclusion of a small short-range charge-dependent nucleon–nucleon potential is necessary to explain the discrepancies between theory and experiment.

2. Calculations

The location of the $T = 1$ level in each of the successive $T = 0$ nuclei is known experimentally (Ajzenberg-Selove *et al.* 1959, 1968, Lauritsen *et al.* 1966, Garvey *et al.* 1964, Wilkinson 1964, Endt *et al.* 1967). These data were used, together with the mass table given by Mattauch *et al.* (1965), to determine the masses of the three members of

each triplet. The values of $b(A, T)$ and $c(A, T)$ were obtained as a function of mass number. The results are given in column (1) of tables 1 and 2 respectively.

Table 1. Results for $b(A, T)$

A	(1)	(2)
6	$-0{\cdot}387 \pm 0{\cdot}053$	$-0{\cdot}305$
10	$-1{\cdot}525 \pm 0{\cdot}032$	$-1{\cdot}509$
12	$-1{\cdot}996 \pm 0{\cdot}096$	$-1{\cdot}902$
14	$-3{\cdot}493 \pm 0{\cdot}052$	$-2{\cdot}385$
16	$-2{\cdot}439 \pm 0{\cdot}026$	$-2{\cdot}329$
18	$-3{\cdot}050 \pm 0{\cdot}011$	$-2{\cdot}987$
24	$-4{\cdot}164 \pm 0{\cdot}032$	$-3{\cdot}823$
28	$-4{\cdot}362 \pm 0{\cdot}027$	$-4{\cdot}231$
32	$-5{\cdot}645 \pm 0{\cdot}138$	$-5{\cdot}598$

(1) $b(A, T)$ from experimental data (MeV); (2) $b(A, T)$ on the basis of Coulomb perturbation (MeV).

Table 2. Results for $c(A, T)$

A	(1)	(2)
6	$0{\cdot}336 \pm 0{\cdot}038$	0·478
10	$0{\cdot}339 \pm 0{\cdot}027$	0·423
12	$0{\cdot}257 \pm 0{\cdot}048$	0·312
14	$0{\cdot}336 \pm 0{\cdot}017$	0·408
16	$0{\cdot}189 \pm 0{\cdot}037$	0·235
18	$0{\cdot}309 \pm 0{\cdot}009$	0·423
24	$0{\cdot}166 \pm 0{\cdot}025$	0·288
28	$0{\cdot}171 \pm 0{\cdot}087$	0·253
32	$0{\cdot}348 \pm 0{\cdot}017$	0·395

(1) $c(A, T)$ from experimental data (MeV); (2) $c(A, T)$ on the basis of Coulomb perturbation.

Theoretically the coefficients were evaluated in the following way. Assuming that Coulomb forces can be treated as a perturbation, then, to first order, the masses of the members of an isobaric multiplet may be written as

$$M(A, T, T_z) = M_0(A, T) + \langle TT_z|H_c|TT_z\rangle + T_z\Delta m$$

where

$$\begin{aligned} H_c &= \frac{e^2}{4r_{12}}\{1-\tau_z(1)\}\{1-\tau_z(2)\} \\ &= \frac{e^2}{4r_{12}}[\{1+\tfrac{1}{3}\boldsymbol{\tau}(1)\,.\,\boldsymbol{\tau}(2)\}+\{-(\tau_z(1)+\tau_z(2))\} \\ &\quad +\{\tau_z(1)\tau_z(2)-\tfrac{1}{3}\boldsymbol{\tau}(1)\,.\,\boldsymbol{\tau}(2)\}] \end{aligned}$$

wherein the terms are so grouped that they form irreducible tensors of rank 0, 1 and 2 in isospin space.

Applying Wigner–Eckart's theorem and comparing the resultant equation with the isobaric mass formula, we obtain

$$b(A, T) = -E^{(1)}_{coul}(A, T) + \Delta m \tag{1}$$

$$c(A, T) = 3E^{(2)}_{coul}(A, T) \tag{2}$$

where the vector and tensor energies $E^{(1)}_{\text{coul}}(A, T)$ and $E^{(2)}_{\text{coul}}(A, T)$ are independent of T_z.

Further, by inverting the equation (Janecke 1966)

$$\langle TT_z|H_c|TT_z\rangle = E_{\text{coul}}(A, T, T_z)$$

$$= E^{(0)}_{\text{coul}}(A, T) - T_z E^{(1)}_{\text{coul}}(A, T) + \{3T_z{}^2 - T(T+1)\}E^{(2)}_{\text{coul}}(A, T)$$

we obtain for $T = 1$

$$E^{(1)}_{\text{coul}}(A, T) = \tfrac{1}{2}[\{E_{\text{coul}}(A, 1, -1) - E_{\text{coul}}(A, 1, 0)\} + \{E_{\text{coul}}(A, 1, 0) - E_{\text{coul}}(A, 1, 1)\}] \quad (3)$$

$$E^{(2)}_{\text{coul}}(A, T) = \tfrac{1}{2}[\{E_{\text{coul}}(A, 1, -1) - E_{\text{coul}}(A, 1, 0)\} - \{E_{\text{coul}}(A, 1, 0) - E_{\text{coul}}(A, 1, 1)\}]. \quad (4)$$

Thus from equations (3) and (4) we see that both vector and tensor Coulomb energies can be expressed in terms of the Coulomb displacement energies between neighbouring members of an isobaric triplet.

In order to derive the Coulomb displacement energies between a pair of isobaric analogue states of neighbouring T_z, it is necessary to calculate the Coulomb interaction of the protons in the unclosed shell with all the protons in the closed shells, together with the mutual Coulomb interaction of the protons in the unclosed shell. The Coulomb energy of the closed shell of protons is assumed to be the same for both states and does not enter into the calculation.

The interaction of a proton in a j' shell with a closed j shell is (de-Shalit and Talmi 1961)

$$\Delta E(j^{2j+1}j'; J = j') = (2j+1)\frac{\sum_{J'}(2J'+1)\Delta E(jj'J')}{\sum_{J'}(2J'+1)}.$$

This is summed for all the closed shells. The interaction energy between n protons in the same j shell is given by (de-Shalit and Talmi 1961)

$$E(j^n) = \tfrac{1}{2}n(n-1)\left\{\frac{2(j+1)\bar{V}_2 - V_0}{2j+1}\right\} + \left[\frac{n}{2}\right]\frac{2(j+1)}{2j+1}(V_0 - \bar{V}_2)$$

with

$$V_0 = \left\langle j^2J = 0\left|\frac{e^2}{4r_{12}}\right|j^2J = 0\right\rangle$$

$$\bar{V}_2 = \frac{\sum_{\substack{J>0\\ \text{even}}}(2J+1)\langle j^2; J|(e^2/4r_{12})|j^2; J\rangle}{\sum_{\substack{J>0\\ \text{even}}}(2J+1)}$$

and $[n/2] = \frac{1}{2}n$ for even n and $\frac{1}{2}(n-1)$ for odd n. Using these formulae the Coulomb displacement energies between neighbouring members of each triplet were calculated. Thus the vector and tensor energies, and consequently the values of $b(A, T)$ and $c(A, T)$, were evaluated as a function of mass number. The matrix elements were calculated using harmonic oscillator wave functions. The oscillator parameter $\alpha = (\hbar/m\omega)^{1/2}$ was determined in each case from the observed radii in electron-scattering experiments (Landolt–Börnstein 1967).

3. Charge-dependent effects

The formulae for b and c are based on the quadratic form of the isobaric mass formula which has been derived on the basis of charge independence of nuclear forces. Now, if the

charge dependence of nuclear forces is of a two-body character and can be treated as a perturbation in the same way as the Coulomb force, then the above quadratic form still holds.

During the last few years several attempts (Blin-Stoyle *et al.* 1962, 1965) have been made to tie down the charge dependence of the internucleon potential. A phenomenological charge-dependent potential has the form

$$H_n = V_0[(p+r\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\{\tau_z(1)+\tau_z(2)\}+(q+s\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\{\tau_z(1)\tau_z(2)-\tfrac{1}{3}\boldsymbol{\tau}(1) \cdot \boldsymbol{\tau}(2)\}] \times \exp\left(-\frac{r_{12}^2}{\mu^2}\right)$$

where V_0 is the typical strength of the charge-independent internucleon potential. This potential is the most general static central two-body charge-dependent potential that can be written down (apart from radial dependence) and for convenience V_0 is taken to be -50 Mev and μ is taken to be 1·73 fm. The parameters p, q, r and s then measure the magnitude of charge dependence.

Since $\tau_z(1)+\tau_z(2)$ is an irreducible tensor of rank 1 in isospin space, we obtain nine linear equations, involving p and r, corresponding to the observed values of $b(A, T)$. Similarly, $\tau_z(1)\tau_z(2)-\frac{1}{3}\boldsymbol{\tau}(1)$, $\boldsymbol{\tau}(2)$ being an irreducible tensor of rank 2 in isospin space, we obtain nine linear equations, involving q and s, corresponding to the observed values of $c(A, T)$. Applying the method of least squares for the solution of linear equations, we obtain the most probable value for the parameters as

$$p = (1{\cdot}983 \pm 0{\cdot}056) \times 10^{-3}$$
$$q = -(2{\cdot}976 \pm 0{\cdot}082) \times 10^{-2}$$
$$r = (1{\cdot}705 \pm 0{\cdot}078) \times 10^{-3}$$
$$s = -(3{\cdot}93 \pm 0{\cdot}056) \times 10^{-2}.$$

4. Conclusions

A comparison between the experimental and theoretical results given in columns (1) and (2) of tables 1 and 2 shows discrepancies when only Coulomb interaction and neutron–proton mass difference are taken into account and charge-dependent effects are ignored.

The above analysis indicates that charge symmetry, i.e. when $p = r = 0$, holds to a greater degree of accuracy than charge independence, i.e. when $p = q = r = s = 0$. The values obtained for the parameters are well within the limits quoted by Blin-Stoyle *et al.* (1965) and Yap (1967). The former have been obtained from a comparison of the binding energies of ^{3}H and ^{3}He, while the latter have been obtained from an analysis of beta decays.

Nevertheless, the above analysis is subject to certain uncertainties. The effect of a small charge-dependent term which arises owing to the difference in the magnetic moment of the neutron and proton has not been taken into account. The calculations have been made assuming that all the nuclei involved are spherical and the large discrepancy between theory and experiment in $A = 24$ is due to the rotational character of the ^{24}Mg nucleus.

The scattering-length difference for proton–proton and neutron–proton scattering in the singlet state indicates that we can certainly expect the nuclear forces to be charge dependent to some extent. It should be noted that there is an important difference between nuclear and Coulomb interactions in the nucleus. The latter are long range, whereas the former are short range, and cancellation of the effects due to the two interactions appears unlikely.

Thus we conclude that a small charge-dependent nuclear perturbation is necessary to explain the energy differences between the members of isobaric multiplets, though the quantitative strength of this charge dependence is still subject to a few uncertainties.

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